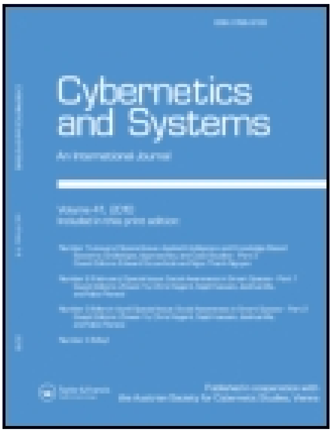

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Igor N. Dubina[a] & Nikolai M. Oskorbin[b]

[a] Institute of Economics, Management and Information Systems, Altai State University, Barnaul, Russia

[b] School of Mathematics and Information Technologies, Altai State University, Barnaul, Russia

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Game-Theoretic Models of Incentive and Control Strategies in Social and Economic Systems

IGOR N. DUBINA[1] and NIKOLAI M. OSKORBIN[2]

[1]*Institute of Economics, Management and Information Systems, Altai State University, Barnaul, Russia*

[2]*School of Mathematics and Information Technologies, Altai State University, Barnaul, Russia*

This study classifies and analyzes incentive systems in social and economic spheres on the methodological basis of game theory. The proposed approach formalizes the assumption of the impossibility of precise performance in socioeconomic realms, which is fundamental for designing incentive systems. The suggested approach addresses latent strategic variables that characterize the behavior of active elements of incentive systems. The authors define, formalize, and characterize authoritarian and democratic systems and mechanisms of incentive and control and demonstrate that a democratic system can significantly reduce incentive and control costs.

KEYWORDS *control, game theory, incentive, optimization*

INTRODUCTION

Incentive and control are key interlinked managerial functions that suppose and ensure proper implementation of tasks (decisions, rules, norms) in economic and social systems. (In order to avoid any confusion, note that we use the word "control" in this article in terms of "checking," not in terms of "regulation" or "command.") The effectiveness of these interrelated functions is very important for the quality of social and economic life. What makes people act most perfectly (i.e., act in an optimal or a feasibly suboptimal

Address correspondence to Igor N. Dubina, Altai State University, Institute of Economics, Management and Information Systems, Sotsialisticheskiy pr. 68, Barnaul 656048, Russia. E-mail: igor_dubina@yahoo.com; din@asu.ru

way) in this or that social, economic, or business situation is a really pivotal question.

Modern incentive and control systems are multilevel systems (i.e., they are usually based on a hierarchy). The presence of several subjects that purposefully influence a dynamical object leads to game theory settings for a control problem (Ugol'nitskii and Usov 2013).

A recent, but already canonical, work on incentive and control theory by Laffont and Martimort (2002) reviews approaches to the problems of incentives during the past two centuries, starting with Adam Smith, and focuses on a game-theoretic, principal-agent model, wherein a principal (a manager or a company) delegates a task to an agent through a contract and tries to maximize the profit (utility) for the company. Another well-known approach to incentive theory and practice is a way to find a compromise (some kind of equilibrium) by threats and counter threats (Moulin 1982; Moulin and Laffond 1981). One more promising, though less well-known, approach we use in this study for the consideration of the problem of incentives and control in hierarchical social and economic systems is the theory of active systems that was established in Russia (Gubko and Novikov 2002).

Based on these approaches, as well as on a number of other works that consider the relations of participants in hierarchical systems (Antipin 2012; Aoki 1976; Chidambaram 1970; Deissenberg and Hartl 2005; Neck 2001; Tanter 1972), we construct a new game-theoretic model of incentive in multilevel hierarchical social and economic systems. This model is developed with an assumption of the impossibility of precise performance in socioeconomic realms, which we consider a fundamental principle for designing incentive and control strategies. Another new element of our approach is addressing latent strategic variables that characterize the behavior of active elements of organizational systems.

With the suggested model, we define, formalize, and examine different types of incentive and control mechanisms in systems with active participants. In particular, we define, characterize, and compare the effectiveness of authoritarian and democratic systems and mechanisms of incentive and control.

BASIC ASSUMPTIONS AND PRINCIPLES OF THE FORMALIZATION OF INCENTIVE SYSTEMS

In many social and economic systems, there is a special and structurally independent (relatively autonomic) element that controls (checks) the performance of the systems' participants. We denote such an element as control authority (CA). A system operator (SO) that is responsible for the whole system function delegates to the CA the functions of incentives and control (checking) over n $(n \geq 1)$ active elements (AE), AE_i $(i = 1, \ldots, n)$, which perform tasks appointed by the SO. Many social and economic systems are organized under such a scheme (e.g., payment control in public transportation systems, tax control, law and social control of individuals, organizations, and social groups).

It is possible to distinguish two polar types of incentive systems: (1) systems with "authoritarian" incentives in which socioeconomic interests of the participants are not considered (A-systems), and (2) systems with "democratic" incentives in which parameters are optimized with the consideration of interests of their elements (D-systems).

Authoritarian systems require clearly formulated objectives and strictly regulated processes when the choice of determined performance parameters is sufficient for the task implementation. Assigned tasks are supposed to be implemented exactly in accordance with clearly defined criteria. Failure detection is followed by punishment ("severe penalties") and the punishment level is so high that the system agents make an effort to exactly perform the assigned task. Each task is supposed to be controlled for effective functioning of such a system. However, in practice, the process of task implementation is controlled selectively.

An alternative way to stimulate agents' performances involves a complex of rewards and punishments under which an active element can choose its level of task implementation. We conventionally designate such a system as a democratic system of incentives.

D-systems are mainly studied and modeled in this article; however, some important aspects of A-systems can be also studied in a similar way. Mathematical formalization of authoritarian and democratic incentive systems is given below.

Let SO set a number of key parameters of a controlled system, as well as standards and regulations of the monitoring process in the organization. These parameters cannot be changed by AEs during their activities and decision-making processes. According to variants of the relation and interaction of SO, CA, and AE_i, the following cases can be defined:

1. SO pays remuneration to CA from a special fund. CA makes decisions for the benefit of the whole system. The relations of SO and CA are established according to the D-system scheme.
2. SO pays remuneration to CA from the amounts of penalties paid by AEs. The relations of SO and CA are established according to the A-system scheme.

The behavior principles of AEs are based on the hypothesis of rational choice and imply the following alternatives:

1. None of AE_i (i=1,…,n) establishes coalitions (with other AEs or CA).
2. Cooperative strategies and coalitions are possible in the system.

In this article, we study the (I–II) class of relations according to this simple classification, and we do not consider information dynamics and asymmetry

(Harsanyi 1995). Thus, we apply the techniques of noncooperative nonantagonistic games with the assumption of infinite repetition of strategy choice under binding agreements in the same environment (Moulin 1982).

Describing the behavior of the performers (AEs), we formalize the following effect: ideal performance (absolutely accurate execution of assigned tasks) is not possible in social and economic systems. This statement is fundamental for our studies, so we will explain it in more detail. In technical systems and manufacturing processes, allowable values of controlled parameters are set through intervals or limits. Such a requirement is easily and naturally assumed by engineers or managers. However, this requirement, as a rule, is not supposed for social, economic, and political systems in order to simplify regulations. Moreover, comparatively small errors made by "loyal" performers are often dissembled, but deviations from assigned norms or rules can always be found in any decision of the opposition party or performers. There is a saying on this occasion: "Under a microscope, even a polished surface seems like the Himalayan Mountains." So, we suppose that the impossibility of ideal performance is essential for studying and designing incentive systems in social and economic spheres.

We will start our study of systems with one AE ($n=1$) and provide some generalization in a concluding section of this article.

A BASIC GAME-THEORETIC MODEL

We formalize and model AE's behavior in a system of type (1-I) with the assumption that AE is entrusted to evaluate parameters of some controlled process (or a controlled object) $v_c \in R^m$, and AE evaluates the parameters (the state of the controlled object) as $v \in R^m$. It means that AE makes an error

$$\varepsilon = \|v - v_c\|, \; v \in V \subset R^m, \tag{1}$$

where ε is some kind of a "distance" between points ν_c and ν in an m-dimensional space.

An aggregate value of AE's diligence in form (1) is also valid in the case when ν_c is the planned performance level and ν is the achieved level. Some flexibility in the assessment of the level of AE's diligence can be provided by a choice of the "distance" value in model (1), i.e., by the measure of discrepancy between the planned and actual levels of the task implementation. In a management sense, $v \in V$ is a real strategy of AE (the achieved level of its performance). It is possible to substitute AE's strategy with an equivalent strategy of AE's choice of ε value in our model. Rationales and conditions of such substitution are discussed in the concluding part of this article.

Some penalty is normally set in order to maintain AE's diligence. We define this penalty with a twice-differentiable function $\psi(\varepsilon)$, which satisfies the following conditions:

$$\psi(0) = 0;\ \psi'_\varepsilon(\varepsilon) \geq 0;\ \psi''_\varepsilon(\varepsilon) \geq 0;\ \exists\varepsilon^0 > 0 : \psi(\varepsilon^0) > 0. \tag{2}$$

Therefore, we demand that the penalty function does not decline with increasing ε and does not take zero value on the whole domain of its definition.

The task implementation supposes AE's labor cost depending on ν value. AE minimizes overall costs by the choice of ε values. A simple model of AE's behavior may be represented by the following AE's objective function:

$$f(\varepsilon) = \psi(\varepsilon) + \varphi(\varepsilon) \to \min_{0\leq\varepsilon\leq\bar{\varepsilon}}. \tag{3}$$

The first term in Eq. (3) is penalty for inefficiency, the second term is AE's cost function, $\bar{\varepsilon} > 0$ is a limit level of the difference between the planned and actual levels of the task implementation. If $\varepsilon \geq \bar{\varepsilon}$, SO implements the rules according to A-system.

Regarding the function $\varphi(\varepsilon)$, we assume that it is twice differentiable, strictly concave down, its values are small at high levels of ε, and it increases significantly for small ε. These properties can be mathematically expressed as follows:

$$\varepsilon \to \infty \Rightarrow \varphi(\varepsilon) \to 0;\ \varepsilon \to 0 \Rightarrow \varphi(\varepsilon) \to \infty;\ \varphi'_\varepsilon(\varepsilon) \leq 0;\ \varphi''_\varepsilon(\varepsilon) > 0. \tag{4}$$

The negative value of derivative $\varphi_\varepsilon'(\varepsilon)$ corresponds to real processes of task implementation in which AE's performance cost decreases with increasing magnitude of ε; i.e., the worse AE's performance is, the less its labor costs. Equation (4) models, in particular, the effect of impossibility of ideally (precisely) performing a planned task.

If conditions (2) and (4) are satisfied, function $f(\varepsilon)$ in Eq. (3) is strictly concave down, and only the solution of $\psi_\varepsilon'(\varepsilon) + \varphi_\varepsilon'(\varepsilon) = 0$ is rooted at point ε^{ST}. So, the solution of Eq. (3) is as follows:

$$\varepsilon^* = \min\left(\varepsilon^{ST};\ \bar{\varepsilon}\right). \tag{5}$$

AE is supposed to act freely and rationally, so it has "a right for some inefficiency," although it has to pay penalty $\psi(\varepsilon^*)$ for such inefficiency. Therefore, when AE decides between strategies, it assesses and compares the level of penalty and its costs. In A-systems, AE should provide just the sufficient accuracy of task implementation at the level specified by administrative regulations. Economic interests and conditions of (3) can be ignored by AE in that case. When we apply and interpret model (3), we must not forget that assessment of penalty is subjective. Consequently, performers' behaviors in the social and economic systems of type D is determined not only by organizational and economic conditions and regulations, but also by psychological, social, cultural, and other factors.

Let $p \in [0, 1]$ be the frequency (probability) of control. The value of p is chosen by CA and is known by AE. If AE has no information on the inspection sequence in an infinitely repeated game, it can determine its choice via the expected value of total costs:

$$f(p, \varepsilon) = p \cdot \psi(\varepsilon) + \varphi(\varepsilon), \;\; p \in [0, 1]. \tag{6}$$

Therefore, a basic mathematical model of an incentive system can be formulated as a two-person nonantagonistic game:

$$f_{CA}(p, \varepsilon) \to \min_{0 \le p \le 1}; \tag{7}$$

$$f(p, \varepsilon) = p \cdot \psi(\varepsilon) + \varphi(\varepsilon) \to \min_{0 \le \varepsilon \le \bar{\varepsilon}}. \tag{8}$$

This model complies with (1-I) system requirements according to the classification provided. The interests of CA in Eq. (7) are characterized with function $f_{\mathrm{CA}}(p, \varepsilon)$ associated with costs of control processes and losses of the system caused by AE's inefficiency. It is natural to assume that CA's costs are zero when control is not carried out ($p = 0$), the task is precisely performed ($\varepsilon = 0$), and the costs monotonically increase when these variables' values grow, i.e.,

$$f_{\mathrm{CA}}(0, 0) = 0; \quad f'_{\mathrm{CA}p}(p, \varepsilon) \ge 0; \quad f'_{\mathrm{CA}\varepsilon}(p, \varepsilon) \ge 0. \tag{9}$$

Let $f_{\mathrm{CA}}(p, \varepsilon)$ be continuous for all of its arguments and specify a function that satisfies this assumption and can be easily interpreted:

$$f_{\mathrm{CA}}(p, \varepsilon) = c \cdot p + d \cdot \varepsilon^2, \;\; c, \; d > 0.$$

In order to simplify the further examination and interpretation of the game in Eqs. (7), (8), we define the functions of costs and penalty in the following game:

$$f_{\mathrm{CA}}(p, \varepsilon) = c \cdot p + d \cdot \varepsilon^2 \to \min_{0 \le p \le 1}, \quad c, d > 0; \tag{10}$$

$$f(p, \varepsilon) = p \cdot a \cdot \varepsilon + \frac{b}{\varepsilon} \to \min_{0 \le \varepsilon \le \bar{\varepsilon}}, \quad a, b > 0. \tag{11}$$

The selected functions satisfy conditions (2), (4), (9). In this case, the solution of $p \cdot \psi_\varepsilon{}'(\varepsilon) + \varphi_\varepsilon{}'(\varepsilon) = 0$ is unique: $\varepsilon^{ST} = \sqrt{\frac{b}{p \cdot a}}$.

EXAMINATION OF THE BASIC MODEL

We will examine game-theoretic model (7), (8) under the assumption of continuity and differentiability of the model functions, as well as assumptions (2), (4), and (9). This examination is carried out by solving the game in terms of prudent (in our case, min-max) strategies, Nash equilibrium, Stackelberg equilibrium, and Pareto optimality. When we interpret the results, we will use special functions in model (10), (11).

Definition 1. Prudent (min-max) strategies in game (7), (8) are the coordinates of point (p^M, ε^M), at which the outer extremes are achieved:

$$L_{\mathrm{CA}} = \min_{0\le p\le 1}\max_{0\le\varepsilon\le\bar{\varepsilon}} f_{\mathrm{CA}}(p,\varepsilon); \quad L = \min_{0\le\varepsilon\le\bar{\varepsilon}}\max_{0\le p\le 1} f(p,\varepsilon), \tag{12}$$

where L_{CA}, L are the guaranteed (upper) values of the players' costs in this game.

Proposition 1. *The min-max strategies in game (7), (8) are* $(p^M = 0, \varepsilon^M = arg \min\limits_{0\le\varepsilon\le\bar{\varepsilon}} f(1,\varepsilon))$, *where* ε^M *is determined according to (5).*

Definition 2. A Nash equilibrium in game (7), (8) is point (p^R, ε^R), at which the following inequalities hold:

$$f_{\mathrm{CA}}(p^R,\varepsilon^R) \le f_{\mathrm{CA}}(p,\varepsilon^R)\ \forall p\in[0,1];\ \ f(p^R,\varepsilon^R) \le f(p^R,\varepsilon)\ \forall\varepsilon\in[0,\bar{\varepsilon}].$$

Proposition 2. *Point* $(0,\bar{\varepsilon})$ *is a Nash equilibrium in game (7), (8).*

The proof of Propositions 1 and 2 are elementary, so we do not dwell on it. We just note that these propositions remain correct for more general penalty functions than the function defined by conditions (2).

In the found equilibrium, frequency of control is zero and the level of performance corresponds to the threshold level in A-systems. Therefore, the examined system implements authoritarian incentive mechanisms, which cannot be used in D-systems. Authoritarian methods of incentives and control are most easily implemented and they can be effective for relatively small $\bar{\varepsilon}$. The found strategic solutions for CA and AE additionally explain the "survivability" of such mechanisms in social and economic systems. It can be naturally expected that the authoritarian methods of incentives and control will be used in managing certain organizational systems (e.g., at the time of combat operations). However, there are alternative mechanisms of incentives and control as well.

Let us assume that the search and implementation of CA and AE's strategies require negotiation processes and concerted collective actions. We will find a game solution in terms of min-max Stackelberg equilibrium

(Jungers et al. 2011) and Pareto-optimal strategies, the stability of which is ensured through threats and counter threats (Moulin and Laffond 1981).

Definition 3. A min-max Stackelberg equilibrium in game (7), (8), where CA is a principal player, is point $(p^{SH}, \varepsilon^{SH})$:

$$p^{SH} = \arg \min_{0 \le p \le 1} \max_{\varepsilon \in R(p)} f_{CA}(p, \varepsilon); \quad \varepsilon^{SH} \in R(p^{SH}), \tag{13}$$

where $R(p) = \mathrm{Arg} \min_{0 \le \varepsilon \le \bar{\varepsilon}} f(p, \varepsilon)$, $p \in [0, 1]$ is a set of optimal strategies of AE (its best response to values of control frequency).

In order to simplify and improve the interpretability of model analysis results, we reformulate game (10), (11) as follows:

$$f_{\mathrm{CA}}(p, \varepsilon) = d(\beta \cdot p + \varepsilon^2) \to \min_{0 \le p \le 1}, \tag{14}$$

$$f(p, \varepsilon) = a(p \cdot \varepsilon + \frac{\alpha}{\varepsilon}) \to \min_{0 \le \varepsilon \le \bar{\varepsilon}}, \tag{15}$$

where α, β are aggregate parameters of the decisions of AE and CA, respectively:

$$\alpha = \frac{b}{a}, \ \alpha \in (0, \infty); \beta = \frac{c}{d}, \ \beta \in (0, \infty).$$

Proposition 3. *Min-max Stackelberg equilibria in game (14), (15) and players' payoffs are as shown in Table 1.*

Proof. Since $R(p) = \mathrm{Arg}\min_{0 \le \varepsilon \le \bar{\varepsilon}} f(p, \varepsilon)$ is a singleton set for each value $p \in [0, 1]$ in this case, it is easy to get $\varepsilon^{SH} = \min\left(\sqrt{\frac{\alpha}{p}};\ \bar{\varepsilon}\right)$.

When $\varepsilon^{SH} = \bar{\varepsilon}, p^{SH} = \arg\min_{0 \le p \le 1}(c \cdot p + d \cdot \bar{\varepsilon}^2)$, we get

$$p^{SH} = 0; \ f_{\mathrm{CA}} = d \cdot \bar{\varepsilon}^2; f = \frac{a \cdot \alpha}{\bar{\varepsilon}}.$$

Thus, the statements on line 3 of Table 1 are proven.

TABLE 1 Players' Optimal Strategies and Payoffs in Game (14), (15)

	p^{SH}	ε^{SH}	$f_{CA}(p^{SH}, \varepsilon^{SH})$	$f(p^{SH}, \varepsilon^{SH})$	Conditions
1	$\sqrt{\frac{\alpha}{\beta}}$	$\sqrt[4]{\alpha \cdot \beta}$	$2d\sqrt{\alpha \cdot \beta}$	$a\sqrt{\alpha}\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt[4]{\alpha \cdot \beta}}$	$\sqrt{\alpha \cdot \beta} \le \bar{\varepsilon}^2; \frac{\alpha}{\beta} \le 1$
2	1	$\sqrt{\alpha}$	$d(\beta + \alpha)$	$2a\sqrt{\alpha}$	$\alpha \le \bar{\varepsilon}^2; \frac{\alpha}{\beta} > 1$
3	0	$\bar{\varepsilon}$	$d \cdot \bar{\varepsilon}^2$	$\frac{a \cdot \alpha}{\bar{\varepsilon}}$	$\alpha > \bar{\varepsilon}^2$

If $\varepsilon^{SH} = \sqrt{\frac{\alpha}{p}}$, we can find p^{SH} from the solution of $p^{SH} = \arg\min_{0 \le p \le 1} d\left(\beta \cdot p + \frac{\alpha}{p}\right)$. We have, consequently, $p^{SH} = \min\left(\sqrt{\frac{\alpha}{\beta}};\ 1\right)$. If $p^{SH} = 1$, then $\varepsilon^{SH} = \sqrt{\alpha}$. Conditions of this solution are $\sqrt{\alpha} \le \bar{\varepsilon}$; $\sqrt{\frac{\alpha}{\beta}} > 1$. The substitution of this solution into the players' payoff functions proves the statements on line 2 of Table 1.

If $p^{SH} = \sqrt{\frac{\alpha}{\beta}}$, then $\varepsilon^{SH} = \sqrt{\frac{\alpha}{p^{SH}}} = \sqrt{\frac{\alpha\sqrt{\beta}}{\sqrt{\alpha}}} = \sqrt[4]{\alpha \cdot \beta}$. The conditions of this solution are $\varepsilon^{SH} \le \bar{\varepsilon}$ and $\sqrt{\frac{\alpha}{\beta}} \le 1$. The substitution of this solution into the players' payoff functions proves the statements on line 1 of Table 1. Thus, Proposition 3 is proven as a whole. □

This result shows that, under the conditions specified in lines 1 and 2 of Table 1, the controlled system operates on the principle of balance of economic interests and without recourse to authoritarian methods. The result of line 3 of Table 1 can be extended to games with any CA's payoff functions that satisfy (9). If $R(p) = \{\bar{\varepsilon}\}$ for all $p \in [0,\ 1]$, then $p^{SH} = 0, \varepsilon^{SH} = \bar{\varepsilon}$, i.e., the system operates in the regime of authoritarian control. We define the relationships, rules, and strategies of the players in this case of optimality in term of the Stackelberg equilibrium as M-1 incentive mechanism.

Let us find Pareto-optimal strategies for the players in game (14), (15). In this case, it is sufficient to specify a set of payoffs (a negotiation set) and the players' threat strategies, considering CA a leading player. The payoffs trajectory in term of Pareto optimality can be found as a solution of

$$f^S_{\mathrm{CA}} = \min_{\substack{0 \le p \le 1 \\ \varepsilon \in [0,\bar{\varepsilon}]}} f_{\mathrm{CA}}(p,\varepsilon);\ f(p,\varepsilon) = f^S;\ f^S \le L; f^S_{\mathrm{CA}} \le L_{\mathrm{CA}}, \tag{16}$$

where L_{CA}, L are guaranteed payoffs of CA and AE (12).

Threat strategies $\tilde{p}_N(\varepsilon),\ \varepsilon \in (0,\bar{\varepsilon}]; \tilde{\varepsilon}_N(p),\ p \in [0,1]$ used by the players in the case of the violation of the agreement $(p^S,\ \varepsilon^S)$ can be determined by the evaluation of the guaranteed results from (12).

Definition 4. A negotiation set is essential if it is not empty and contains more than two different elements.

Proposition 4. *In game (14), (15), a negotiation set is essential and has a form as follows:*

$$S = \{(f_{CA}, f) : f_{CA} = \frac{d \cdot a^2 \cdot \alpha^2}{f^2};\ \frac{a \cdot \alpha}{\bar{\varepsilon}} \le f \le L\}. \tag{17}$$

For any pairs of $(f_{CA}^S, f^S) \in S$, *the threat strategies are constant functions* $\tilde{p}_N(\varepsilon) = 1,\ \varepsilon \in (0, \bar{\varepsilon}]; \tilde{\varepsilon}_N(p) = \bar{\varepsilon},\ p \in [0, 1]$, *and the negotiation strategies are*

$$p^S = 0,\ \varepsilon^S = \frac{a \cdot \alpha}{f^S}.$$

The players use the following rules to choose their strategies:

$$CA: \quad \tilde{p}(\varepsilon) = \begin{cases} 0, & \text{if } \varepsilon = \varepsilon^S \\ 1, & \text{otherwise} \end{cases}; \tag{18}$$

$$AE: \quad \tilde{\varepsilon}(p) = \begin{cases} \varepsilon^S, & \text{if } p = 0 \\ \bar{\varepsilon}, & \text{otherwise} \end{cases}. \tag{19}$$

Proof. The guaranteed results and threat strategies in game (14), (15), according to (12), are as follows:

$$L_{\mathrm{CA}} = d \cdot \bar{\varepsilon}^2; L = \min\left(2a\sqrt{\alpha};\ a(\bar{\varepsilon} + \frac{\alpha}{\bar{\varepsilon}})\right); \tilde{p}_N(\varepsilon)$$
$$= 1,\ \varepsilon \in (0, \bar{\varepsilon}]; \tilde{\varepsilon}_N(p) = \bar{\varepsilon},\ p \in [0, 1].$$

Model (16) for game (14), (15) is as follows:

$$f_{\mathrm{CA}}^S = \min_{\substack{0 \le p \le 1 \\ \varepsilon \in [0, \bar{\varepsilon}]}} d\,(\beta \cdot p + \varepsilon^2);\ a\,(p \cdot \varepsilon + \frac{\alpha}{\varepsilon}) = f^S.$$

The solution is $p^* = 0,\ \varepsilon^* = \frac{a \cdot \alpha}{f^S}$; therefore $f_{\mathrm{CA}}^S = \frac{d \cdot a^2 \cdot \alpha^2}{(f^S)^2}$.

Figure 1 illustrates this negotiation set.

The lower value of AE's payment is $f_N = \sqrt{\frac{d \cdot a^2 \cdot \alpha^2}{L_{\mathrm{CA}}}} = \frac{a \cdot \alpha}{\bar{\varepsilon}}$.

Set S is essential if $f_N < L$. Direct calculation shows the fair of inequality $f_N < L$. If we assume $\sqrt{\alpha} < \bar{\varepsilon}$, we get $L = 2a\sqrt{\alpha}$. Strict inequality $\frac{a \cdot \alpha}{\bar{\varepsilon}} < 2a\sqrt{\alpha}$ is fair when $\frac{1}{2}\sqrt{\alpha} < \bar{\varepsilon}$; that is consistent with the original inequality $\sqrt{\alpha} < \bar{\varepsilon}$.

Let us assume $\sqrt{\alpha} \ge \bar{\varepsilon}$ and $L = a(\bar{\varepsilon} + \frac{\alpha}{\bar{\varepsilon}})$. Inequality $\frac{a \cdot \alpha}{\bar{\varepsilon}} < a\,(\bar{\varepsilon} + \frac{\alpha}{\bar{\varepsilon}})$ is true, since the limit value of penalty $a \cdot \bar{\varepsilon}$ is strictly positive. Thus, Proposition 4 is proven. □

In the context of the discussion of the obtained results, let us note that the maximum value of penalty $a \cdot \bar{\varepsilon}$ characterizes the negotiation range not just for AE, but also for CA. With the assumption of $\sqrt{\alpha} < \bar{\varepsilon}$, CA's payoff at the point H (Figure 1) is $f_{\mathrm{CA}}^H = \frac{d \cdot \alpha}{4}$ as a consequence of the proof of Proposition 4. The upper value of CA's costs is equal to the guaranteed result $L_{\mathrm{CA}} = d \cdot \bar{\varepsilon}^2$. From the difference of these values $\Delta = (d \cdot \bar{\varepsilon}^2 - \frac{d \cdot \alpha}{4}) >$

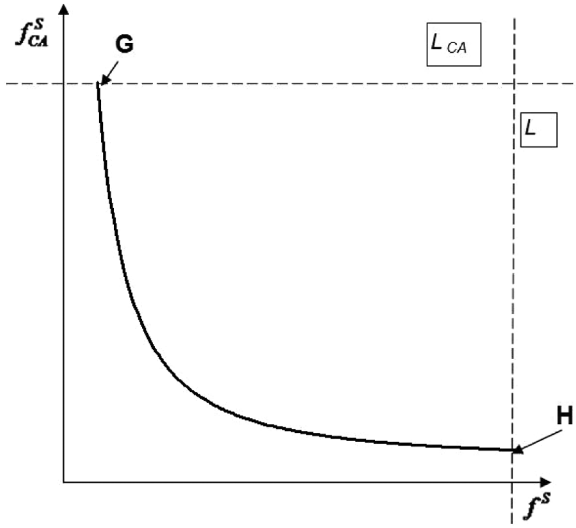

FIGURE 1 A negotiation set in game (14), (15).

$(d \cdot \bar{\varepsilon}^2 - \frac{d \cdot \bar{\varepsilon}^2}{4}) = \frac{3d \cdot \bar{\varepsilon}^2}{4}$, it follows that CA can significantly (about 75%) reduce the loss of SO by using stable Pareto-optimal strategies. This value might vary depending on the outcome of the negotiation process and the value of α (the integral characteristics of AE). This effect is achieved through the cooperation of CA and AE, when CA does not demand penalty and AE increases their work effectiveness. Thus, the principle of "accuracy for a penalty waiver" is implemented in this organizational system.

Expression (19) can be interpreted for social systems as a variant of protest movement of employees, who, as a rule, do not initiate a negotiation process.

Expression (18) arouses the following concerns: (i) whether the equality $\varepsilon = \varepsilon^S$ can be achieved and (ii) how CA can control this equality. In the context of the analyzed system (class 1-I), it is enough for CA to require inequality $\varepsilon \leq \varepsilon^S$ in Eq. (18). Thus, AE, according to the principle of rational choice, perceives this inequality as a threshold limit and chooses its optimal strategy (ε^S). An incentive mechanism based on the requirement of inequality $\varepsilon \leq \varepsilon^S$ has preventing character for AE who checks an actual performance value ε^R against ε^s.

Let us consider a way of indicator control for inequality $\varepsilon \leq \varepsilon^S$ required to implement strategy (18). Such indicator control should be effective both cost-wise and operationally. Thus, an additional system of indicator control might be required for the implementation of strategy (18). Examples of such systems are tax service and defaulter identification in public transportation in some countries. Recently, video footage and information technologies are widely used for these purposes. In many countries with a developed system of social partnership, people are actively engaged in indicator control under social rules implementation.

The stability of Pareto-optimal strategies depends on negotiation results and the balance of CA and AE's interests. In order to explicitly express the achieved balance level, we define the value of f^S, which determines ε^S in (18) as follows:

$$f^S = L(1-\gamma),$$

where γ is the level of concessions.

When $\gamma = 0$, AE's payment is equivalent to its guaranteed result, and the negotiations complete by the choice of point H, as indicated in Figure 1. It is possible to calculate the upper limit of $\bar{\gamma} < 1$ at which negotiation results move to point G. The choice of $\gamma \in (0, \bar{\gamma})$ is not defined as part of our formalization of organizational systems, so it can be considered an external independent parameter. With these comments, the chosen strategy of CA can be written as follows:

$$\tilde{p}(\varepsilon) = \begin{cases} 0, & \text{if } \varepsilon \leq \varepsilon^S = \frac{a \cdot \alpha}{L(1-\gamma)} \\ 1, & \text{otherwise} \end{cases} \tag{20}$$

We define a mechanism that implements the described system of relations and strategy (20) as M-2 incentive and control mechanism.

A NUMERICAL ILLUSTRATION AND INTERPRETATION

For purposes of the interpretation of CA and AE's strategies, as well as the comparison of M-1 and M-2 mechanisms, we provide a numerical example. Input (assumed) data and calculated results of this example are presented in Table 2. We assume that $\bar{\varepsilon}$ is large enough and corresponds to the conditions of the first line of Table 1. Thus, we examine interaction and negotiations of the control system participants. With a significant level of concessions ($\gamma = 0.4$) in this control system, CA's total cost for M-2 is almost 11 times less than its cost for M-1 (Table 2, line 14). This effect is achieved through the cooperation principle of "accuracy for a penalty waiver" implemented in M-2. In practice, this effect may be somewhat reduced because of additional cost of monitoring of possible AE deviations from a cooperative strategy.

An example of the use of M-1 mechanism is tax control organization in some countries (including Russia), where each accounting transaction is directly or indirectly verified by tax inspectors. However, it is not possible to identify all legislation violations because of corruption relations and the "goodwill" of officials, so actual control probability is less than one ($p^{SH} < 1$). Low efficiency and costly nature of tax control with M-1 confirm our conclusions. Similar mechanisms are also used for control of activities of

TABLE 2 Input Data and Results of M-1 and M-2 Comparison.

	Parameter	Symbol	Control Mechanism M-1	Control Mechanism M-2
Input Date				
1	Penalty coefficient	a	100	100
2	AE's cost function parameter	b	1	1
3	AE characteristics	α	0.01	0.01
4	CA's losses function parameter	d	7	7
5	Control cost coefficient in CA's losses function	c	1	1
6	CA characteristics	β	1/7	1/7
7	Level of concessions	γ	0	0.4
Optimal Strategies of the Players				
8	Control frequency	p^{SH}; p^S	0.26	0
9	Diligence (accuracy)	ε^{SH}; ε^S	0.194	0.083
Results of Analysis of the Control System				
10	Penalty value	ψ	5.04	0
11	AE costs	ϕ	5.15	12.00
12	AE payoff	f	10.19	12.00
13	AE guaranteed result	L	20.00	20.00
14	CA payoff	f_{CA}	0.529	0.049

government and public and private organizations in many countries. "Vitality" of such mechanisms can be particularly explained by a low level of organizational culture and historical factors.

An example for the implementation of M-2 is a system of public law support, in which M-1 cannot be principally realized. Law enforcement of individual violations is initiated either by the injured party, or by a special public service. The effectiveness of M-2 mechanism strongly depends on validity of the established norms (strategy ε^S) and an employed system of indicator control and the realization of threat and punishment strategies.

Another problem of M-2 implementation in social and economic systems relates to legal recognition and formulation of impossibility of the perfect (ideal) task implementation and, consequently, admissibility of errors within $\varepsilon < \varepsilon^S$. An example of such a problem is the effort to set permissible blood alcohol content for car drivers. Russian legislation has repeatedly established and annulled such rules in recent years, but social consensus has not yet been reached yet (ITAR-TASS 2013; Kononova 2010).

This aspect of control should be studied further in detail. In our view, two general approaches can be defined. The first approach is to establish clear and easily controllable indicators of norms violation (for example, disaffection in communication with a policeman). The second approach is to establish aggregative behavior norms ("rough rules"), which, when followed, would create the effect of ideal behavior. This does not exclude setting acceptable standards in accordance with theoretical recommendations when possible.

MODEL GENERALIZATION AND CONCLUDING REMARKS

In order to generalize the suggested models, their interpretations and applications, we consider a game-theoretic model of a system of (1-I) type with n active elements (AEs):

$$f_{CA}(p,\varepsilon) = \sum_{i=1}^{n} f_{CAi}(p_i,\varepsilon_i) \to \min_{0\le p_i\le 1,\ i=1,\dots,n}; \tag{21}$$

$$f_i(p_i,\varepsilon_i) = p_i\cdot\psi_i(\varepsilon_i) + \varphi_i(\varepsilon_i) \to \min_{0\le\varepsilon_i\le\bar{\varepsilon}_i}; \quad i=1,\dots,n. \tag{22}$$

This model assumes that each AE$_i$ ($i=1,\dots,n$) controls the level of its diligence (accuracy) $\varepsilon_i\in[0,\bar{\varepsilon}_i]$, and CA sets its strategy of incentives and control for each AE$_i$. Game (21), (22) can be decomposed into n independent games, so the results obtained for the previous models are valid for this case as well. In particular, the following proposition generalizes Proposition 4 and can be similarly proven.

Proposition 5. *In game (21), (22) a negotiation set is essential and has the following form:*

$$S_i = \{(f^S_{CAi}, f^S_i) : f^S_{CAi} = f_{CAi}(0,\varepsilon^S_i);\ f^S_i = \varphi_i(\varepsilon^S_i); f^S_i \le L_i; f^S_{CAi} \le L_{CAi}\}.$$

Internal variable ε^S_i is the solution of equation $\varphi_i(\varepsilon_i) = f^S_i$, which is uniquely solvable for all values $f^S_i > 0$ upon conditions (4). For any pairs of $(f^S_{CAi}, f^S_i)\in S_i$, the threat strategies are constant functions $\tilde{p}_{Ni}(\varepsilon_i) = 1$, $\varepsilon_i\in(0,\bar{\varepsilon}_i]$; $\tilde{\varepsilon}_{Ni}(p) = \bar{\varepsilon}_i$, $p_i\in[0,1]$.

Another general comment relates to the constraints for $p\in[0, 1]$. In some cases, the upper value of frequency control $\bar{p}$ ($p\in[0,\bar{p}]$) can be strictly less than 1 due to operational or financial constraints. The results obtained previously are completely valid in this case as well. It is enough to substitute just a variable $p\in[0,\bar{p}]$ with a dummy variable of frequency control $q\in[0, 1]$ in the proposed game-theoretic models. However, this substitution must be also considered for identification of the model parameters and interpretation of modeling results.

Finally, let us consider the substitution of strategy $v\in V$ with strategy $\varepsilon\in[0,\bar{\varepsilon}]$ according to (1). Validity of this substitution can be proven in the frame of properties of systems with latent variables that have the following structure:

$$y = f(\tilde{z}(v)),\quad v\in V\subset R^m;\ \tilde{z}(v):\ V\to Z. \tag{23}$$

In this case, latent variables ν determine the controllable variables z, but do not directly affect the output values y. Such systems are constructed, for example, when we model socioeconomic processes in which variables z are controlled on the upper level of the system (by CA in our case), and values of latent variables (hidden for the upper level) are chosen by active components of the system (AEs in our case; Dubina 2013; Oskorbin 2012). Thus, a game of CA and AE can be formulated as follows:

$$y_{\mathrm{CA}} = f_{\mathrm{CA}}(z^0, \tilde{z}(v)) \ \rightarrow \max_{z^0 \in Z^0}; \tag{24}$$

$$y_{\mathrm{AE}} = f_{\mathrm{AE}}(z^0, \tilde{z}(v)) \ \rightarrow \max_{v \in V}, \tag{25}$$

where y_{CA} and y_{AE} are payments of CA and AE correspondingly, z^0 is a vector of controllable strategic variables of CA and ν is a vector of latent strategic variable of AE. The solution of game (24), (25) is based on its equivalent transformation in accordance with (23).

Thus, CA's objective function is

$$y_{\mathrm{CA}} = f_{\mathrm{CA}}(z^0, \tilde{z}(v)) \ \rightarrow \max_{z^0 \in Z^0}, \tag{26}$$

AE's objective function is

$$y_{\mathrm{AE}} = f_{\mathrm{AE}}(z^0, z) \ \rightarrow \max_{z \in Z}, \tag{27}$$

and AE's auxiliary task is

$$\delta = \min_{v \in V} \|z^* - \tilde{z}(v)\| = 0. \tag{28}$$

Economic sense of this game is as follows. CA and AE "play" on the sets of controllable variables of the system (z^0 and z), since (26), (27) do not depend on the latent variables if players know set Z. After finding a compromise solution (z^{0*}, z^*), AE chooses optimal values of its latent variables according to (28) and thus ensures the implementation of its decision $z^* = \tilde{z}(v^*)$, where v^* is optimal behavior of AE. This approach can be applied to any number of active elements and, thus, grounds the substitution of strategy $v \in V$ with strategy $\varepsilon \in (0, \bar{\varepsilon}]$, since game (7), (8) is structurally a particular case of game (26), (27), (28). At the same time, it is assumed that expression (1) meets the following requirements: V is a nonempty, closed and bounded subset of R^m ; $v_c \in V$; $\max_{v \in V} \|v - v_c\| \geq \bar{\varepsilon}$.

So, in this article, we have made an attempt to formalize relations and interactions of the participants of a hierarchical organizational system whose objectives and functions are summarized in Table 3.

Generally, this research defines an approach to designing effective incentives and control mechanisms in different types of multilevel organizational systems. However, the basic model proposed and studied in this article is

TABLE 3 Functional Structure of a Formalized Hierarchical System.

Participants	Objectives and Functions
System Operator (SO)	• Evaluating the system parameters • Setting a task • Setting the level of responsibilities of CA • Designing a control system • Allocating resources for supporting control system operations • Evaluating the effectiveness of the control system
Control Authority (CA)	• Selecting incentive and control mechanisms • Designing and organizing incentives and control processes • Coordinating with SO
Active Elements (AEs)	• Evaluating of the assigned task and the conditions for its implementation • Decision-making on the performance level • Organizing and monitoring the assigned task implementation • Coordinating with CA

mostly theoretical. For practical applications, it is necessary to develop applied models and mechanisms in order to identify their parameters.

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