Wavelet Transform Modeling of a Short Electromagnetic Pulse Scattering by Multilayered Structure

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Abstract—This paper considers short electromagnetic pulse scattering by multilayered dielectric structure. Incident field is generated by thready source of electric current, which is located parallel to the boundaries of layers. The solution is presented with Fourier transform on spatial coordinates and wavelet transform on time coordinate.

Keywords—wavelet transform, pulse, scattering, dielectric layer.

I. INTRODUCTION

Studying propagation and scattering of electromagnetic fields by local objects represents significant interest in problems of remote sensing, radar-location, geolocation. [1] Furthermore actual problems of technical diagnostics are using electromagnetic fields what makes short electromagnetic pulse scattering problems popular [1-5].

Theoretical researches of electromagnetic pulses propagation and scattering problems raise hardship with the choice of solution method. The main requirement to the method is taking into account the pulse character of the source. The conventional techniques based on Fourier or Laplace transforms are unsuitable on numerous occasions due to difficulties of solution interpretation or numerical calculations. So, it is necessary to find the method considering the local nature of incident field. The candidate is to apply the adaptive integral wavelet transform through the time coordinate.

In this paper the electromagnetic pulse scattering by multilayered dielectric structure problem is considered. The incident field is excited by the thin line source of electric current.

II. PROBLEM FORMULATION

Infinitely long along axis \( y \) line source of electromagnetic surface current density \( j' \) is located in \( (x_0, z_0) \) point. Incident horizontal polarization field is scattered by multilayered dielectric structure with layer thickness \( d_i \), where \( i \) is the layer number from 2 to \( M - 1 \). Relative dielectrical permittivity of the halfspaces and structure layers are \( \varepsilon_i \). Magnetic permeability is equal to 1 everywhere. The problem is two-dimensional, so \( \frac{\partial}{\partial y} = 0 \).

Time dependence function of the source is:

\[
j(t) = -\frac{t - t_u}{\tau^2} e^{-\frac{(t - t_u)^2}{2\tau^2}},
\]

where \( \tau \) – pulse half-width. Incident field falls to the dielectric structure and partially reflects and passes inside enduring multiple reflections.

Fourier transform by spatial coordinates and continuous wavelet transform by time coordinate have been used:
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\[ W(\Omega, T) = \int_{-\infty}^{+\infty} f(t) \sqrt{\omega} \psi(\Omega(t - T)) dt \]

\[ f(t) = \frac{1}{C} \int_{-\infty}^{+\infty} W(\Omega, T) \sqrt{\omega} \psi(\Omega(t - T)) d\Omega dT. \]  

Normalization constant \( C \) is given by equation:

\[ C = \int_{-\infty}^{+\infty} |\psi(\omega)|^2 d\omega, \]

where \( \psi(\omega) \) is Fourier image of the first order Gauss wavelet function \( \psi(t) \).

III. ELECTROMAGNETIC FIELDS AND BORDER CONDITIONS

Maxwell’s equations of the problem are divided into two independent equations systems with \( (E_x, H_x, H_z) \) electromagnetic components of horizontal polarization and \( (E_y, E_z, H_y) \) components of vertical polarization. The solution for the case of horizontal polarization is considered in this paper.

To construct the solution let’s take advantage of the similar problem results treated in [4] for the dual layer structure. Time wavelet transform and spatial Fourier transform representation of electromagnetic fields in \( z > 0 \) halfspace can be written in the following form:

\[ E_x(x, z, t) = \frac{1}{C(2\pi)^{3/2}} \int_{-\infty}^{+\infty} e^{i \xi \cdot \eta} \psi_{\text{str}}(\xi, \eta, \Omega, T) \frac{d\xi \cdot \eta}{2 \pi} \]

\[ H_x(x, z, t) = \frac{1}{C(2\pi)^{3/2}} \int_{-\infty}^{+\infty} h_{\text{str}}(\xi, \eta, \Omega, T) \frac{d\xi \cdot \eta}{2 \pi} \]

\[ H_y(x, z, t) = \frac{1}{C(2\pi)^{3/2}} \int_{-\infty}^{+\infty} h_{\text{str}}(\xi, \eta, \Omega, T) \frac{d\xi \cdot \eta}{2 \pi} \]

Spectral densities of the incident fields:

\[ \psi_{\text{str}}(\xi, \eta, \Omega, T) = -\mu, I^r(\xi, \eta, \Omega, T) \cdot \psi_{\text{str}}(\xi, \eta, \Omega, T) \]

\[ \psi_{\text{str}}(\xi, \eta, \Omega, T) = -\mu, H^r(\xi, \eta, \Omega, T) \cdot \psi_{\text{str}}(\xi, \eta, \Omega, T) \]

\[ H_{\text{str}}(\xi, \eta, \Omega, T) = \eta_i \psi_{\text{str}}(\xi, \eta, \Omega, T) \]

\[ W(\Omega, T) = \int_{-\infty}^{+\infty} f(t) \sqrt{\omega} \psi(\Omega(t - T)) dt \]

\[ f(t) = \frac{1}{C} \int_{-\infty}^{+\infty} W(\Omega, T) \sqrt{\omega} \psi(\Omega(t - T)) d\Omega dT. \]

After integrating by \( \eta \) coordinate and using residue theorem expressions for the incident fields take the form:

\[ E_x(x, z, t) = \frac{1}{4\pi C} \int_{-\infty}^{+\infty} \frac{i \psi_{\text{str}}(\xi, \eta, \Omega, T) \xi \cdot \eta}{2 \pi} \frac{d\xi \cdot \eta}{2 \pi} \]

\[ H_x(x, z, t) = \frac{1}{4\pi C} \int_{-\infty}^{+\infty} \frac{\psi_{\text{str}}(\xi, \eta, \Omega, T) \xi \cdot \eta}{2 \pi} \frac{d\xi \cdot \eta}{2 \pi} \]

\[ H_y(x, z, t) = \frac{1}{4\pi C} \int_{-\infty}^{+\infty} \frac{\psi_{\text{str}}(\xi, \eta, \Omega, T) \xi \cdot \eta}{2 \pi} \frac{d\xi \cdot \eta}{2 \pi} \]

\[ \psi_{\text{str}}(t) = \frac{d\psi_{\text{str}}(t)}{dt} \]

Denoting delay operator \( \hat{D}(x) : f(t) \rightarrow f(t - x) \) border condition for \( z = 0 \) can be presented as follows:

\[ \begin{cases} 
\hat{D}(t_e) E_x^+ + E_x^- = E_x' \\
\hat{D}(t_e) H_y^+ + H_y^- = H_y' 
\end{cases} \]

where \( t_e = \frac{x^2 + z^2}{c^2} \).

Denote operators \( \hat{L}_{\text{str}, N} \) of the transition from layer \( n \) to the neighbor layer with \( N \) inner reflections, upper index shows resulting field direction. Correspondingly, \( \hat{L}_{\text{str}, N} \) operator determines the transition from layer \( n \) to the neighbor layer with infinite inner reflections:

\[ E_x(x, z, t) = \frac{1}{4\pi C} \int_{-\infty}^{+\infty} \frac{i \psi_{\text{str}}(\xi, \eta, \Omega, T) \xi \cdot \eta}{2 \pi} \frac{d\xi \cdot \eta}{2 \pi} \]

\[ H_x(x, z, t) = \frac{1}{4\pi C} \int_{-\infty}^{+\infty} \frac{\psi_{\text{str}}(\xi, \eta, \Omega, T) \xi \cdot \eta}{2 \pi} \frac{d\xi \cdot \eta}{2 \pi} \]

\[ H_y(x, z, t) = \frac{1}{4\pi C} \int_{-\infty}^{+\infty} \frac{\psi_{\text{str}}(\xi, \eta, \Omega, T) \xi \cdot \eta}{2 \pi} \frac{d\xi \cdot \eta}{2 \pi} \]

\[ \psi_{\text{str}}(t) = \frac{d\psi_{\text{str}}(t)}{dt} \]

\[ \begin{cases} 
\hat{D}(t_e) E_x^+ + E_x^- = E_x' \\
\hat{D}(t_e) H_y^+ + H_y^- = H_y' 
\end{cases} \]
\[\begin{align*}
\mathbf{F}_{\text{inc},x,l,N}^x &= \hat{D}(t_x + 2Nt_i)T_{x,l,N}R_{x,l,N}^x e^{i(k_x-x_{l,N})d_x}, \\
\mathbf{F}_{\text{inc},z,l,N}^z &= \hat{D}(t_z + 2Nt_i)T_{z,l,N}R_{z,l,N}^z e^{i(k_z-z_{l,N})d_z}, \\
\mathbf{F}_{\text{inc},l,1,N}^z &= \hat{D}(t_z + 2Nt_i)T_{l,1,N}R_{l,1,N}^z e^{i(k_z-z_{l,1,N})d_z}, \\
\mathbf{F}_{\text{inc},l,1,N}^x &= \hat{D}(t_x + 2Nt_i)T_{l,1,N}R_{l,1,N}^x e^{i(k_x-x_{l,1,N})d_x}, \\
\mathbf{L}_{x,l,N}^z &= \sum_{x=1}^{N} \mathbf{F}_{\text{inc},x,l,N}^z, \\
\mathbf{L}_{z,l,N}^x &= \sum_{x=1}^{N} \mathbf{F}_{\text{inc},z,l,N}^x, \\
\mathbf{L}_{l,1,N}^x &= \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^x, \\
\mathbf{L}_{l,1,N}^z &= \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^z, \\
e_l^x &= e_l^x(\xi_{l,N},\Omega,T,t) = \frac{-i\mu_0}{w_1} e^{i\omega_{l,N}c_0}, \\
J(\Omega,T) &= \sqrt{\mu_0} w_{l,N}(t), \\
R_x &= \frac{w_x - w_{x-1}}{w_x + w_{x-1}} = \frac{2w_x}{w_x + w_{x-1}}, R_z = R_x, R_{z,x} = e^{i2\pi n_{l,N}}. \\
\end{align*}\]

Let's analyze electromagnetic field propagation through the set of dielectric layers. At first, the incident field sequentially penetrates into the all layers of the structure.

\[\begin{align*}
e_{e,x}(\xi_{l,N},\Omega,T,z,t) &= \hat{D} \left( t_x - \frac{z d_x}{c_x} \right), \\
e_{e,x}(\xi_{l,N},\Omega,T,z,t) &= 0. \\
\end{align*}\]

Then there are multiple inner reflections generated inside every layer:

\[\begin{align*}
e_{e,x}(\xi_{l,N},\Omega,T,z,t) &= \hat{D} \left( t_x - \frac{z d_x}{c_x} \right), \\
\mathbf{L}_{z,l,N}^x &= \sum_{x=1}^{N} \mathbf{F}_{\text{inc},x,l,N}^z, \\
e_{e,x}(\xi_{l,N},\Omega,T,z,t) &= \hat{D} \left( t_x + \frac{z d_x}{c_x} \right). \\
\end{align*}\]

Next reflections fields transit into all surrounding layers:

\[e_{e,x}(\xi_{l,N},\Omega,T,z,t) = \sum_{x=1}^{N} \mathbf{F}_{\text{inc},x,l,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},z,l,N}^x(\xi_{l,N},\Omega,T,t) + e_{e,x}(\xi_{l,N},\Omega,T,z,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^z(\xi_{l,N},\Omega,T,t) + e_{e,x}(\xi_{l,N},\Omega,T,z,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^z(\xi_{l,N},\Omega,T,t). \tag{12}\]

After that there are multiple inner reflections generated inside every layer again:

\[\begin{align*}
e_{e,x}(\xi_{l,N},\Omega,T,z,t) &= \sum_{x=1}^{N} \mathbf{F}_{\text{inc},x,l,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},z,l,N}^x(\xi_{l,N},\Omega,T,t) + e_{e,x}(\xi_{l,N},\Omega,T,z,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^z(\xi_{l,N},\Omega,T,t) + e_{e,x}(\xi_{l,N},\Omega,T,z,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^z(\xi_{l,N},\Omega,T,t). \tag{13}\]

Finally, we get recurrent relations:

\[\begin{align*}
e_{e,x}(\xi_{l,N},\Omega,T,z,t) &= \sum_{x=1}^{N} \mathbf{F}_{\text{inc},x,l,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},z,l,N}^x(\xi_{l,N},\Omega,T,t) + e_{e,x}(\xi_{l,N},\Omega,T,z,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^x(\xi_{l,N},\Omega,T,t) + \sum_{x=1}^{N} \mathbf{F}_{\text{inc},l,1,N}^z(\xi_{l,N},\Omega,T,t). \tag{14}\]

Moreover, the final expression for the required field components and their spectral densities can be represented as follows:

\[e_{e}(\xi_{l,N},\Omega,T,z,t) = \frac{\hat{D} e_l^x(\xi_{l,N},\Omega,T,t)}{4\pi c}, \quad E_s(x,z,t) = \frac{1}{4\pi c} e_l^x(\xi_{l,N},\Omega,T,z,t), \quad e^{i\omega_{l,N}c_0} + c_l^x(\xi_{l,N},\Omega,T,z,t), \quad e^{i\omega_{l,N}c_0} + e_l^x(\xi_{l,N},\Omega,T,z,t). \tag{16}\]
Expressions for magnetic fields components can be obtained using (16). Relations for the bottom halfspace can be derived from (16) for $M - 1$ layer using transition operators (9). Electric field intensity expressions (16) contains infinite sums that determine reflections in the layers and the influence of surrounding layers. Every reflection and transition between the layers contributes time delay depending on layer width. So due to pulse limitation we could estimate the tile delay of the every sum term and limit the number of necessary calculating terms. Some numerical results for the case of $M = 4$ were obtained in [4,5].

IV. CONCLUSION

The problem of a short electromagnetic pulse scattering by multilayered dielectric structure using adaptive integral transform by time coordinate and Fourier transform by spatial coordinates has been solved. Expressions for required electromagnetic fields components in the different space regions has been obtained.

REFERENCES


